Evolution of shower parton distributions in a jet from quark recombination model

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The evolution of shower parton distributions in a jet is investigated in the framework of quark recombination model. The distributions are parameterized and the Q^2 dependence of the parameters is given by polynomials of $\ln Q^2$ for a wide range of Q^2.

I. INTRODUCTION

The study of hadron production in high energy collisions is the only way to understanding structure of hadrons and the dynamics of the collision process. Because of asymptotic freedom at short distance and quark confinement at long distance of the strong interaction, hadron formation is a soft process which cannot be calculated perturbatively from the first-principle for the strong interactions–the quantum chromodynamics (QCD). Therefore some model considerations are needed whenever hadron formation is involved in a process. One of the most frequently used models for hadron production is the parton fragmentation model. This model was first proposed in studying the hadron production in e^+e^- annihilation process [1]. In such a process, e^+e^- annihilate into a virtual photon which then materializes into a $q\bar{q}$ pair. The produced q and $\bar{q}$ move away from each other and each fragments or cascades into a collection of hadrons (or jet). Fragmentation functions have been used to parameterize the non-perturbative aspects of jets from initiating quarks and gluons in last a few decades. As stated clearly in [2], however, this approach "is not meant to be a theory. It is simply a parameterization that incorporates many of the expected features of fragmentation." The fragmentation functions does not tell us how the hard partons fragment into jets. If hard parton fragmentation were the only way for the production of hadrons with large transverse momentum, the ratio between two hadron species, for any given transverse momentum, would be a process independent constant dictated by the fragmentation functions. However, experimental data at the Relativistic Heavy Ion Collider (RHIC) at Brookhaven have shown that the proton to pion ratio can be about 1 for $p_T \sim 3$ GeV/c in central AuAu collisions at $\sqrt{s_{NN}} = 200$ GeV [3], much larger than the corresponding value in pp collisions. This experimental fact indicates that the study of the mechanism of parton fragmentation is necessary in order to understand the colliding system dependence of hadron production in a jet.

A theoretical microscopic description of parton fragmentation is suggested in [4] based on quark recombination approach [5, 6] for hadron production. The physics issue in the microscopic description of fragmentation is that every quark and gluon with high virtuality is assumed to evolve by emanating quark-antiquark pairs and gluons with lower virtualities into a shower of partons with low transverse momentum relative to the initiating parton direction. The shower partons recombine then to form the final state hadrons. In this way, the fragmentation functions can be related to the shower parton distributions in a jet. In high energy hadronic and nuclear collisions, there exist some soft partons with low virtuality and some hard partons produced in hard collisions with high momentum transfer. Those hard partons may have very high virtuality and, according to the principles of QCD, will evolve into parton showers. Those shower partons, together with the soft partons, then recombine into final state hadrons detected in experiments, as described in the recombination model.

In [4] shower parton distributions initiated by a hard parton with virtuality $Q^2 = 100$ (GeV/$c)^2$ were obtained from the fragmentation functions determined by fitting experimental data for high energy e^+e^- annihilation and hadronic collisions. Once the shower parton distributions are obtained, it is natural to apply them in modeling hadron production in other collisions. In [6] it is shown that the recombination of soft partons and shower partons is important in reproducing the transverse momentum spectra of produced particles in both AuAu and dAu collisions and in explaining the centrality dependence of the Cronin effect [7] in dAu collisions.

In this paper, we investigate the parton distributions in a jet for Q^2 from 2 to 1000 (GeV/$c)^2$. We also parameterize the Q^2 dependence of the parameters for the parton distributions for easy use in the recombination model description for hadron productions in different collision processes. The application of the parton distributions will be left for later work. This paper is organized as follows. In section II, we briefly discuss the recombination model description of parton fragmentation. Then in section III, we discuss the Q^2 evolution of the parton distributions. The last section is for a short discussion.

II. SHOWER DISTRIBUTIONS IN A JET FROM RECOMBINATION MODEL

For brevity, one can introduce following notation for any two parton distributions $f_1(x)$ and $f_2(x)$ in a jet,

$$\{f_1 f_2\}(x_1, x_2) \equiv 0.5 \left(f_1(x_1) f_2\left(\frac{x_2}{1-x_1}\right) + \right.$$

$$f_1\left(\frac{x_1}{1-x_2}\right)f_2(x_2)\bigg) \ .$$

Here x_1, x_2 are momentum fractions of two partons relative to the initiating parton. In this expression, kinetic constraint is imposed so that $x_1 + x_2 < 1$ for any pair of two quarks.

In the quark recombination model for fragmentation the generic formula for the production of a meson composed of a quark q and an antiquark $\bar{q}'$ is [4, 8]

$$xD(x) = \int_0^x \frac{dx_1}{x_1}\int_0^x \frac{dx_2}{x_2}\{q\bar{q}'\}(x_1,x_2)R(x_1,x_2,x) \ , \quad (1)$$

where $q(x_1)$ and $\bar{q}'(x_2)$ are the parton distributions of a quark q at momentum fraction x_1 and an antiquark $\bar{q}'$ at x_2 in a jet, and $R(x_1,x_2,x)$ is the recombination function (RF) for the formation of a meson at x from the quark q and antiquark $\bar{q}'$. In the recombination model, the gluons are assumed to convert first to quarks and antiquarks before hadron formation. So in last formula no gluon distribution is considered.
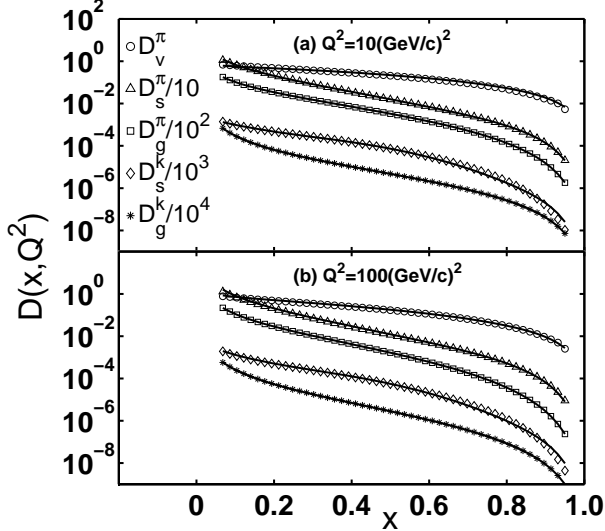

FIG. 1: Fitted fragmentation functions for valence quark to pion, sea quark to pion, gluon to pion, and sea quark to kaon, gluon to kaon, at (a) $Q^2 = 10$ and (b) 100 $(\text{GeV}/c)^2$. The fitted results are shown as solid curves, and the parameterizations from KKP (for gluon jets) and Kretzer (for quark jets) by different symbols for different fragmentation functions.

At this stage, we only consider production of hadrons composed of light quarks $(u, \bar{u}, d, \bar{d}, s, \bar{s})$ fragmented from light hard partons $(u, \bar{u}, d, \bar{d}, s, \bar{s}, g)$. If we assume SU(2) symmetry and symmetric $u\bar{u}$ and $d\bar{d}$ sea quark distributions, there are five independent parton distributions. Three of them are for parton distributions in a jet initiated by light quarks. They are the non-singlet distribution K_{NS} of valence quark with the same flavor as the initiating (anti)quark, the non-strange sea quark distribution L, the strange sea quark distribution L_s. The

other two are for the non-strange light quark distribution G and strange quark distribution G_s in a gluon jet. Five fragmentation functions are needed in order to determine those five parton distributions. We work with fragmentation functions for the pion production D_S^π from sea quarks, D_V^π from valence quark, D_G^π from gluon jet, and for K production D_S^K from sea quarks, D_G^K from gluon jet. The corresponding expressions for the fragmentation functions in terms of the parton distributions are given in the quark recombination model as [4]

$$xD_S^\pi(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\{LL\}(x_1,x_2)R^\pi(x_1,x_2,x), \quad (2)$$

$$xD_V^\pi(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\{KL\}(x_1,x_2)R^\pi(x_1,x_2,x), \quad (3)$$

$$xD_G^\pi(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\{GG\}(x_1,x_2)R^\pi(x_1,x_2,x), \quad (4)$$

$$xD_S^K(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\{LL_s\}(x_1,x_2)R^K(x_1,x_2,x), \quad (5)$$

$$xD_G^K(x) = \int \frac{dx_1}{x_1}\frac{dx_2}{x_2}\{GG_s\}(x_1,x_2)R^K(x_1,x_2,x). \quad (6)$$

In last equations R^π and R^K are RF for pions and kaons, respectively. Here, both the fragmentation functions and the parton distributions in a jet depend on the virtuality Q^2 of the initiating parton.

Before one can start determining the parton distributions from above equations, one needs to make a decision on choosing parameterizations of the fragmentation functions given by different groups. In fact, the determination of fragmentation functions is not as well as supposed to be. Different physics considerations are input in different parameterizations of the fragmentation functions, and sometimes the obtained results are not consistent with each other. The most frequently used parameterizations of the fragmentation functions are given by Kniehl, Kramer, Pötter (KKP) [9, 10] and by Kretzer [11]. In [11] it is assumed that valence quark dominates the production of hadrons with large momentum fraction and sea quarks contribute more to hadron production at small momentum fraction. In KKP parameterizations, no such physics consideration is input, but they used more recent experimental results involving gluons. In our paper, we adopt Kretzer parameterization of the fragmentation functions for jets initiated by quarks or antiquarks for a better consideration of valence quark contributions, but use KKP parameterization for gluon jets.

In fitting the fragmentation functions from quark recombination model, as given in Eqs. (2-6), we assume the parton distributions to be the form

$$f(z) = Az^a(1-z)^b(1+cz^d) \ , \quad (7)$$

with five parameters A, a, b, c and d, and z the momentum fraction of the quark relative to the initiating parton. For the valence quark distribution, a condition is imposed

$$\int \frac{dz}{z} K_{NS}(z) = 1 \ , \tag{8}$$

because the total number of valence quark in a jet cannot be more than one.

The fragmentation functions for pion and kaon have been fixed in [12] as

$$R^{\pi}(x_1, x_2, x) \ = \ \frac{x_1 x_2}{x^2} \delta \left(\frac{x_1 + x_2}{x} - 1 \right) \ , \tag{9}$$

$$R^{K}(x_1, x_2, x) \ = \ 12 \frac{x_1^2 x_2^3}{x^5} \delta \left(\frac{x_1 + x_2}{x} - 1 \right) \ . \tag{10}$$

In the recombination function R^K for kaons, x_2 refers to the momentum fraction of the strange (anti)quark and x_1 to that of the non-strange (anti)quark.

Then one can fit the fragmentation functions and get the parton distributions in a jet for any fixed Q^2. The obtained parameters A, a, b, c and d for the five parton distributions depend on the scale Q^2. For $Q^2 = 10$ and 100 $(\text{GeV}/c)^2$ the fitted results are shown in Fig. 1 as solid curves, in comparison with the KKP and Kretzer parameterizations. Obviously, the recombination model can fit the fragmentation functions very well. It should noted that the fitted curve for K_{NS} agrees with Kretzer parameterization quite well, in sharp difference from the fitted result in [4], there the parameterization for the valence quark fragmentation had a significant contribution at small momentum fraction region, while in Kretzer parameterization the valence quark fragmentation is assumed to dominate in large momentum fraction region and no singularity at very small momentum fraction.

The fitted shower parton distribution functions $(pdf(z))$ are shown in Fig. 2 for $Q^2 = 10$ and 100 $(\text{GeV}/c)^2$. As expected, the non-singlet distribution K_{NS} for valence quark is hardest among the five distributions. Other parton distributions show a strong increase at small z as z goes to smaller values.

III. EVOLUTION OF PARTON DISTRIBUTIONS

In [4] only the fragmentation functions at fixed $Q^2 = 100$ $(\text{GeV}/c)^2$ are employed. The obtained parton distributions are then used to calculate the contributions from both soft-shower and shower-shower parton recombination to the hadron p_T spectra. For simplicity, all hard partons are assumed to have the same virtuality $Q^2 = 100$ $(\text{GeV}/c)^2$ in all applications. In more realistic cases, partons with different momenta may have different virtualities. This difference in virtuality may influence hadron productions from the fragmentation of hard partons and from other quark recombination processes.
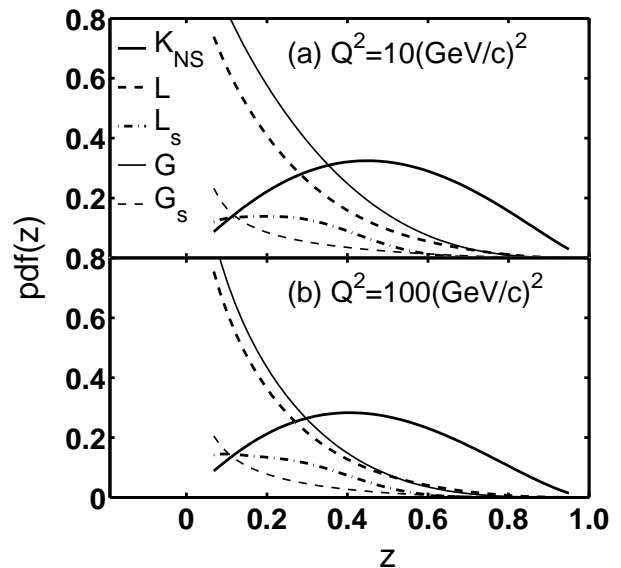

FIG. 2: Fitted shower parton distribution functions (pdf(z)) in a jet at (a) $Q^2 = 10$ and (b) 100 $(\text{GeV}/c)^2$.
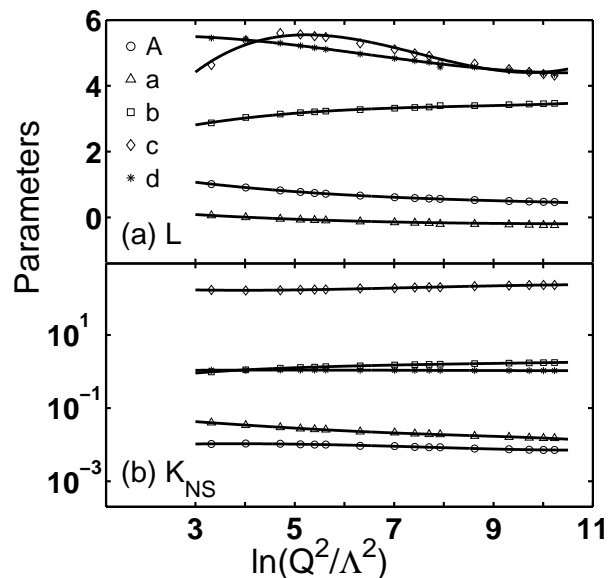

FIG. 3: Q^2 dependence of five parameters for parton distributions K_{NS} and L. Solid curves are fitted results by Eq. (11). The QCD scale Λ is chosen to be 0.19 $(\text{GeV}/c)^2$.

From perturbative QCD we know that the Q^2 evolution of the fragmentation functions is described by the DGLAP equations [13]. From such equations one can determine the fragmentation functions at any Q^2 if one knows them at one given scale Q_0^2. Up to now however, we did not know the Q^2 dependence of parton distributions in a jet initiated by a hard parton, nor have we had an equation to describe their evolution in a model independent way. But we can get such a dependence by using Eqs. (2-6) from the Q^2 evolution of the fragmentation

functions. In this paper, we simply use the FORTRAN code provided by Kretzer [14] to get fragmentation functions at any given Q^2. For each Q^2 we use Eqs. (2-6) to fit the fragmentation functions and get the corresponding parameters for each parton distributions at that scale Q^2.

In accordance with other theoretical results, we define here a dimensionless scale $u = \ln(Q^2/\Lambda^2)$ with the QCD scale $\Lambda = 0.19 \text{GeV}/c$. It is found that the parameters for each shower parton distribution has a strong dependence on the scale u. As examples, the Q^2 dependence of parameters A, a, b, c, d for L and K_{NS}, obtained from fitting the fragmentation functions at different Q^2, are shown in Fig. 3 as functions of $u = \ln(Q^2/\Lambda^2)$ by symbols. For easy use of others, one can express the Q^2 dependence of the parameters by a polynomial of the scale u as

$$A(Q^2) = C_0 u^3 + C_1 u^2 + C_2 u + C_3 . \qquad (11)$$

The fitted results are shown by solid curves also in Fig. 3 for comparison. Then the dependence of every of five parameters for a shower parton distribution in a jet is given by four parameters, C_0, C_1, C_2 and C_3 in a wide range of Q^2 from 2 to 1000 $(\text{GeV}/c)^2$. Since the Q^2 dependence of each parameter in the parton distributions can be well reproduced by only four constant coefficients, Eq. (11) is a quite economic description of the evolution of shower parton distributions. The coefficients C_0, C_1, C_2 and C_3 are tabulated in Table I for all the parameters for the five parton shower distributions discussed in this paper.

From this table, parton distributions can be calculated easily for a jet with any scale Q^2 in the range considered in this paper.

IV. DISCUSSIONS

The evolution of shower parton distributions in a jet is discussed in the framework of quark recombination model from the Q^2 dependence of the fragmentation functions. We show that the evolution of the distributions can be given by the polynomial $\ln Q^2$ dependence of a few parameters. From the tabulated coefficients one can calculate corresponding parton distributions in a jet and other fragmentation functions needed in the framework of quark recombination model. Together with soft and hard parton distributions, these shower parton distributions can be used for the production of hadrons in different collision processes.

Acknowledgments

[1] R.D. Field and R.P. Feynman, Nucl. Phys. **B 136**, 1 (1978).

[2] R.D. Field, *Applications of Perturbative QCD*, Addison-Wesley Publishing Company (1989).

[3] S.S. Alder, PHENIX Collaboration, Phys. Rev. **C 69**, 034909 (2004).

[4] R.C. Hwa and C.B. Yang, Phys. Rev. **C70**, 024904 (2004).

[5] R.C. Hwa and C.B. Yang, Phys. Rev. **C 67**, 034902 (2003); V. Greco, C.M. Ko, and P. Levai, Phys. Rev. Lett. **90**, 202302 (2003); Phys. Rev. **C 68**, 024902 (2003); R.J. Fries, B. Müller, C. Nonaka, and S.A. Bass, Phys. Rev. Lett. **90**, 202303(2003); Phys. Rev. **C 68**, 044902 (2003).

[6] R.C. Hwa and C.B. Yang, Phys. Rev. Lett. **93**, 082302 (2004); Phys. Rev. **C 70**, 024905 (2004); Phys. Rev. **C70**, 037901 (2004); Phys. Rev. **C 70**, 054902 (2004).

[7] J.W. Cronin et al., Phys. Rev. **D 11**, 3105 (1975).

[8] R.C. Hwa, Phys. Rev. **D 22**, 1593 (1980).

[9] B.A. Kniehl, G. Kramer and B. Pötter, Nucl. Phys. **B 582**, 514 (2000).

[10] B.A. Kniehl, G. Kramer and B. Pötter, Nucl. Phys. **B 597**, 337 (2001).

[11] S. Kretzer, Phys. Rev. **D 62**, 054001 (2000).

[12] R.C. Hwa and C.B. Yang, Phys. Rev. **C 66**, 025205 (2002).

[13] Yu. Dokshitzer, Sov. Phys. JETP **46**, 1649 (1977); V.N. Gribov, L.N. Lipatov, Sov. Nucl. Phys. **15**, 438 (1972); V.N. Gribov, L.N. Lipatov, Sov. Nucl. Phys. **15**, 675 (1972); G. Altarelli and G. Parisi, Nucl. Phys. **B 126**, 298 (1977).

[14] See the website http://www.pv.infn.it/~radici/FFdatabase/

TABLE I: Fitted coefficients in Eq. (11) for parameters of the parton distributions in a jet.

		C_0	C_1	C_2	C_3
K_{NS}	A	2.27×10^{-5}	-4.91×10^{-4}	2.78×10^{-3}	5.92×10^{-3}
	a	-7.21×10^{-5}	1.94×10^{-3}	-1.92×10^{-2}	8.46×10^{-2}
	b	1.24×10^{-3}	-3.47×10^{-2}	0.398	-6.24×10^{-3}
	c	-0.328	7.67	-45.3	246
	d	2.92×10^{-4}	-7.17×10^{-3}	4.88×10^{-2}	0.993
L	A	-0.0009	0.0275	-0.3179	1.7997
	a	-0.0003	0.0116	-0.1489	0.4376
	b	0.0023	-0.0588	0.5344	1.6756
	c	0.0251	-0.5642	3.8456	-2.7196
	d	0.0040	-0.0773	0.2938	5.2021
L_s	A	-0.0427	0.9992	-7.7046	21.3488
	a	-0.0049	0.1210	-1.0107	3.5884
	b	-0.0022	0.0442	-0.1268	8.1004
	c	0.1677	-4.2572	38.9198	-18.9039
	d	-0.0014	0.0308	-0.2036	3.4815
G	A	-0.0025	0.0835	-0.9312	3.9350
	a	0.0004	0.0007	-0.1318	0.5742
	b	-0.0015	-0.0030	0.4695	-0.2050
	c	0.0003	-0.0073	0.0575	-1.1224
	d	0.0014	-0.0906	1.0256	-1.6076
G_s	A	-0.0015	0.0319	-0.2183	0.5004
	a	-0.0140	0.3064	-2.0830	3.6414
	b	-0.0061	0.1095	-0.3339	1.6614
	c	0.1277	-2.7241	18.0042	-34.0906
	d	-0.0128	0.2812	-1.9713	5.6142